

# TOWARDS THE NON-CHIRAL EXTENSION OF SM AND MSSM

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## Abstract

We show that in some SU(N) type GUTs with the complementary pairs of the conjugated fermion multiplets there naturally appear the relatively light ( $M \ll M_{GUT}$ ) vectorlike fermions which considerably modify the desert physics. In the non-SUSY case they can provide for the unification of the standard coupling constant  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_S$  whereas in the SUSY case they can increase the unification point up to the string unification limit and decrease  $\alpha_S(M_Z)$  down to the value predicted from the low energy physics.

**Keywords:** Standard Model, SUSY, GUT, RG equation, non-chiral fermions, gauge coupling unification.

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# 1 Introduction

The LEP confirmation [1] of three neutrino species in Z-bozon decays seems to mean that we have in general only three standard chiral quark-lepton families. Actually, there is not in fact a viable way to the heavy ( $m_\nu > M_Z/2$ ) neutrino for the hypothetical fourth family in the framework of the SM to say nothing of the GUTs. So, one can say after observation of the top quark at FERMILAB [2] that all the chiral matter of the SM are already discovered and now only its SUSY counterpart, "smarter" of the MSSM, remains to be detected. Needless to stress specially that not only the MSSM but the ordinary minimal GUTs like as SU(5) or SO(10) [3] and their SUSY versions [4] also suggest the pure chiral extension of the SM.

What one could say now about the fermion matter-fields vectorlike under the SM,  $\Psi_{SM} + \bar{\Psi}_{SM}$ ? If they exist, where could be their mass scale? The common experience tells us that it could be somewhere near the scale of the "next" chiral symmetry manifesting itself generally in the form of  $SM \otimes D$  or  $GUT \otimes D$  or possibly  $D \supset GUT$  where D stands for discrete or global or even local symmetry covering the known minimal GUTs SU(5) or SO(10) (the letter abbreviations such as SM, GUT etc. are used everywhere for the underlying symmetries as well). In contrast to the SM D-symmetry somehow differentiates the left- and right-handed components of the above vectorlike matter-fields and thus protect their masses from being much heavier than its own scale  $V_D$ . The "next" chiral symmetry could be some family symmetry H (say, a chiral  $SU(3)_H$  symmetry acting in 3-dimensional generation space [5]) or Peccei-Quinn symmetry  $U(1)_{PQ}$  [6] both concerning besides the ordinary SM chiral quarks and leptons some vectorlike pairs of the fermion multiplets conjugated under the SM. These multiplets receive their masses of order  $V_H$  or  $V_{PQ}$  after H-symmetry or  $U(1)_{PQ}$  is spontaneously broken. One more example could be suggested by non-minimal GUTs, e.g.  $SU(N > 5)$  [7] which contain generally many vectorlike fermion pairs  $(5 + \bar{5})$  and  $(10 + \bar{10})$  of  $SU(5)$ . In the ordinary exposition [7] their masses are appeared to be about the scale of breaking of the  $SU(N)$  GUT down to  $SU(5)$  and thus have no real influence

even on the near-GUT physics not to mention the low-energy one. However, there could be the other breaking channels as well (not necessarily following through the Georgi-Glashow  $SU(5)$  [3]) giving lower masses for the non-chiral fermions [8]. So, to conclude some additional fermion matter, if it exists, should certainly be vectorlike under SM or even GUT and be accommodated in general somewhere in the grand desert between the SM and GUT scales.

Another question is do we really need any additional fermion matter beyond the SM multiplets of quarks and leptons. The answer could be positive if we wanted to overcome the crisis, related with the actual non-unification of the standard coupling constants  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_S$  in the SM [9]. Also in the MSSM this unification [9] does not look enough conclusive if one keep in mind the real gap between the SUSY GUT and string unification cases [10] as well as some discrepancy for  $\alpha_S(M_Z)$  predicted from the single scale SUSY GUT unification on the one hand and low energy data on the other [11]. It has been shown recently [11] that in the minimal  $SU(5)$  type theories, the inclusion of the threshold corrections does not change this situation. Thus one could expect that some new physics might appear in the grand desert accommodating the above SM vectorlike matter at the certain intermediate scale  $V_D$  so that to improve the running of the standard coupling constants correspondingly.

In this letter we show that along the certain breaking channels of the  $SU(N) \otimes D$  type GUTs extended to include the additional complementary pairs of the  $SU(N)$  conjugated fermion multiplets there naturally appear the relatively light ( $M \ll M_{GUT}$ ) vectorlike fermions depending on a group order and starting multiplets involved. They turn out to considerably modify the desert physics. In the non-SUSY case they provide for the unification of the standard coupling constants, whereas in the SUSY case they can increase the unification point up to the string unification limit and decrease  $\alpha_S(M_Z)$  down to the value predicted from low-energy physics.

## 2 Hyperneutral split fermions

So, we start with a general  $SU(N) \otimes D$  GUT containing besides some "standard" anomaly free set of fermion multiplet with ordinary quarks and leptons

$$3 \cdot \left[ (N - 4) \cdot \bar{\psi} \begin{pmatrix} N \\ 1 \end{pmatrix} + \psi \begin{pmatrix} N \\ 2 \end{pmatrix} \right] \quad (1)$$

the  $n_F$  pairs of the conjugated chiral fermions (complementary fermions)

$$n_F \cdot \left[ \Psi_1 \begin{pmatrix} N \\ K \end{pmatrix} + \bar{\Psi}_2 \begin{pmatrix} N \\ K \end{pmatrix} \right], \quad (2)$$

(in the left-handed basis) where  $\binom{N}{K}$  stands for dimension of the particular  $SU(N)$  antisymmetric representation used merely to have only the quark and lepton type states in  $\Psi_1$  and  $\bar{\Psi}_2$ .

The most natural genesis of the SM from the general  $SU(N)$  theory looks as the spontaneous breakdown of the starting  $SU(N) \otimes D$  symmetry due to the "standard" scalar set which includes one adjoint ( $\Phi_j^i$ ) and  $N - 5$  fundamental ( $\varphi_i^{(r)}$ ) heavy scalars of  $SU(N)$  ( $i, j = 1, \dots, N; r = 1, \dots, N - 5$ )

$$SU(N) \otimes D \xrightarrow{\Phi} SU(n)_S \otimes SU(N - n)_W \otimes U(I) \xrightarrow{\varphi^{(r)}} SU(3)_S \otimes SU(2)_W \otimes U(1) \quad (3)$$

accommodating generally the strong and weak parts of the SM in the different subgroups  $SU(n)_S$  and  $SU(N - n)_W$ , respectively. The natural case when the scalars  $\Phi$  and  $\varphi^{(r)}$  have the same order VEVs ( $\Lambda \sim \lambda^{(r)}$ ) corresponds to the minimal (one-scale)  $SU(N)$  GUTs broken down to the SM below the unification point. It goes without saying that besides  $\Phi$  and  $\varphi^{(r)}$  scalars there are generally two scalar field multiplets  $H_1$  and  $H_2$  which breaks subsequently the SM and give masses to the *up* and *down* quark, respectively.

The simplest choice for chiral D symmetry here in the framework  $SU(N)$  GUTs seems to be the familiar reflection [4] for the adjoint scalar  $\Phi$  accompanied now by appropriate reflection in the vectorlike pairs (2)

$$\Phi \rightarrow -\Phi, \quad \Psi_1 \rightarrow \Psi_1, \quad \bar{\Psi}_2 \rightarrow -\bar{\Psi}_2 \quad (4)$$

so that their direct  $SU(N)$  invariant mass term is fully suppressed and only their Yukawa couplings with scalar  $\Phi_j^i = (\phi^A \cdot T^A)_j^i$

$$G \cdot \bar{\Psi}_2 \Phi \Psi_1 + h.c. \quad (5)$$

( $G$  is a coupling constant and  $T^A$  are generators of  $SU(N)$ ) are allowed to exist. So, the masses of the vectorlike fermions (2) will completely determined by the VEV matrix  $\langle \Phi_j^i \rangle$  only. It is well known [12] that the adjoint scalar  $\Phi$  itself develops VEV along one of the hypercharges  $\hat{Y}_N^{(n,N-n)}$  of  $SU(N)$  providing for the first stage in the breaking chain (3). What this means is the submultiplets in  $\Psi_1$  and  $\bar{\Psi}_2$  which have  $Y_N = 0$  drop out of the basic coupling (5) and thus leave to be massless until the fundamental scalars  $\varphi^{(r)}$  break  $Y_N$  making the adjoint scalar  $\Phi$  develop the VEV along the other hypercharges as well. Actually, the fundamental scalars  $\varphi^{(r)}$  having no the direct Yukawa couplings with  $\Psi_1$  and  $\bar{\Psi}_2$  affect their mass spectrum only through the intersecting terms in the general Higgs potential  $V$  of scalars  $\Phi$  and  $\varphi^{(r)}$

$$V(\Phi, \varphi^{(r)}) = \dots + a(Tr\Phi^2)^2 + b(Tr\Phi^4) + \alpha\bar{\varphi}\varphi Tr\Phi^2 + \beta\bar{\varphi}\Phi^2\varphi + \dots \quad (6)$$

(two last terms in (6), index  $r$  is omitted) inducing in the starting VEV matrix  $\langle \Phi_j^i \rangle$

$$\langle \Phi_j^i \rangle = \Lambda diag \left[ 1\dots 1, -\frac{n}{N-n}, \dots -\frac{n}{N-n} \right]_j^i = \Lambda \left[ Y_N^{n,N-n} \right]_j^i \quad (7)$$

some other the SM invariant corrections of order

$$\epsilon^{(r)} \Lambda, \quad \epsilon^{(r)} = \frac{\beta^{(r)}}{b} \left( \frac{\lambda^{(r)}}{\Lambda} \right)^2 \quad (8)$$

during the second stage of the symmetry breaking process (3). In such a manner the mass matrix of the complementary fermions (2) can be expressed generally through the hypercharges of  $SU(N)$  as

$$\hat{M} = M_0 \cdot \hat{Y}_N + \sum_{r=1}^{N-5} M_s \cdot \hat{Y}_{N-r}, \quad M_0 = G\Lambda, M_r = G\Lambda\epsilon^{(r)} \quad (9)$$

(correct to some calculable group factors in  $M_0$  and  $M_r$ ), where  $\hat{Y}_N$  corresponds to the hypercharge matrix of U(I) in (3) and the others ( $\hat{Y}_{N-r}$ ) belong to  $SU(N-r)$  groups while the last one  $\hat{Y}_5$  is the familiar hypercharge of the standard SU(5) [4] or what is the same the normalized hypercharge of the SM.

One can see now that we are driven at the natural mass-splitting inside of the fermion pairs (2) depending on the U(I) hypercharge values of their  $SU(n)_S \otimes SU(N-n)_W$  submultiplets. While the general mass scale of the  $\Psi$  pairs (2) is given by the largest mass  $M_0$  in Eq.(9) their U(I) neutral submultiplets survive the first stage of the symmetry breaking in (3) and receive masses of the order of  $M_r$  only during subsequent breakings down to the SM. This mass gap in the split  $\Psi$  multiplets (2), even within the soft radiative hierarchy between VEVs  $\lambda^{(r)}$  and  $\Lambda$  or, preferably, between coupling constants  $\beta^{(r)}$  and  $b$  in Eq.(8), appears to strongly affect the running of the standard gauge coupling constants  $\alpha_1, \alpha_2$  and  $\alpha_S$  in the superhigh energy area (see below). Some motivation for  $\beta^{(r)} \ll b$  would stem from that the corresponding coupling in the Higgs potential (6) has the starting Lagrangian (local) symmetry  $SU(N)$  only, whereas the other terms in it are invariant under independent  $SU(N)_\Phi$  and  $SU(N)_\varphi$  transformations as well. Thus the last term in the potential (6) could purely be induced by the  $SU(N)$  gauge loops giving the natural radiative order for constant  $\beta$ ,  $\beta \sim \alpha_{GUT}^2$ . So, even in the case of the single point unification ( $\Lambda \sim \lambda^{(r)}$ ) there would be a few order hierarchy between the mass parameters  $M_0$  and  $M_r$  ( $\frac{M_0}{M_r} \sim \alpha_{GUT}^2 = 10^3 - 10^4$ ) in Eq.(9)<sup>2</sup>. It seems to be quite remarkable that while in that case the  $SU(N)$  GUT breaks down to the SM at once, the  $\Psi$  multiplet mass spectrum still follow to the two-step breaking process (3) generating among others the relatively light masses of the U(I) neutral submultiplets in  $\Psi$ . So, the reflection (4) not only protect the complementary fermions from having the heavy  $SU(N)$  invariant masses but also provide for some their submultiplets the masses much lower then its own scale  $\Lambda$ . We call them the hyperneutral split fermions (HSF).

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<sup>2</sup>Of course, this hierarchy could be considerably enhanced if we suppressed a constant  $\beta$  by some special fine tuning or the SUSY arguments (see below).

### 3 The HSF scanning the SU(N) GUTs

Let us imagine for a moment that we know nothing about the SU(N) GUTs at all and will scan them now depending on the mass spectrum of the complementary fermions only - whether there are allowed to exist the hyperneutral split fermions or not.

Let the complementary fermion multiplets  $\Psi_1$  and  $\bar{\Psi}_2$  (2) belong to some pure antisymmetric representation of SU(N) broken to  $SU(n)_S \otimes SU(N-n)_W \otimes U(I)$ . One can see that the HSF submultiplets will appear if the following group conditions are satisfied

$$\frac{k_n}{n} = \frac{k_{N-n}}{N-n} = \frac{K}{N} \quad (10)$$

where K is the order (number of indices) of the multiplets  $\Psi_1$  and  $\bar{\Psi}_2$  under SU(N) whereas  $k_n$  and  $k_{N-n}$  are suborders of their HSF submultiplets under subgroups  $SU(n)_S$  and  $SU(N-n)_W$ , respectively ( $k_n + k_{N-n} = K$ ). Eq.(10) follows, by definition, from the zero U(I) hypercharge value for the HSF submultiplets as it can be derived by direct application the  $\hat{Y}_N$  matrix (7) to them.

Now using the basic condition (10) we are able to carry out the general classification of all the possible SU(N) GUTs depending upon the order value K only of the complementary fermions:

(i)  $K = 1$  does not lead to any HSF submultiplets. The only solution  $n = N$  of Eq.(10) corresponds to the non-broken  $SU(N)$ ;

(ii)  $K = 2$  gives  $k_n = 2\frac{n}{N} = 1$  (keeping in mind that  $k_n \leq K$  by definition, whereas  $k_n = K$  conforms again with the non-broken symmetry case) and leads to the  $SU(2n)$  GUTs with breaking pattern  $SU(n)_S \otimes SU(n)_W \otimes U(I)$  and the HSF submultiplets in the representation  $(n, n) + (\bar{n}, \bar{n})$  whose decomposition under SM gives

$$\left[ (3_c, 2) + (3_c, 1)_d + h.c. \right] + (n-3) \left[ 5 + \bar{5} \right] \quad (11)$$

apart from the trivial SM singlets. Here index  $d$  means the down quark type state in  $(3_c, 1)$  and  $(\bar{3}_c, 1)$  while  $5$  and  $\bar{5}$  stand for the standard  $SU(5)$  quintet and antiquintet, respectively.

(iii) In the general case  $K = 1, 2, \dots, \frac{N}{2}(\frac{N-1}{2})$  we are led to the  $SU(n + n\frac{k_{N-n}}{k_n})$  GUTs ( $n = 1, \dots, N; k_n = 1, \dots, K; k_n + k_{N-n} = K$ ) with the breaking pattern  $SU(n)_S \otimes SU(n\frac{k_{N-n}}{k_n})_w \otimes U(I)$  and the HSF submultiplets in the representation

$$\left[ \begin{pmatrix} n \\ k_n \end{pmatrix}, \begin{pmatrix} n\frac{k_{N-n}}{k_n} \\ k_{N-n} \end{pmatrix} \right] + h.c. \quad (12)$$

from where the previous particular cases can easily be reproduced.

So, keeping in mind  $N \geq 5$  for the  $SU(N)$  GUT covering the SM we should conclude that a familiar  $SU(5)$  can not satisfy the criterion condition (10) for any non-trivial HSF submultiplet and has been excluded completely. The same occurs for the other prime order  $SU(N)$  GUTs ( $N = 7, 11, \dots$ ) unless as they preliminary break to one of the allowable cases (see (iii)) due to some other mechanism and thereupon follow to the our HSF scenario.

The another point is that among the all above HSF versions the case (ii) seems to be certainly singled out. As one can see from Eq.(11) there appear that all  $SU(2n)$  theories contain in their HSF sectors the common SM fragments plus the  $SU(5)$  full quintets which does not affect (in the leading 1-loop order) the unification picture. Thus this picture could be expected very similar for the whole class of the  $SU(2n)$  GUTs as we found there the practically order-invariant HSF submultiplets.

An inspection of this order-invariant part of the HSF submultiplets (the first term in Eq.(11)) together with two<sup>3</sup> electroweak doublets in the starting Higgs field  $H^1$  and  $H^2$  in the model shows that we have directly driven just at so called ABC split-multiplet Anzatz postulated in the framework of the standard  $SU(5)$  a decade ago by Glashow and Frampton

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<sup>3</sup>One can see that all the GUTs beyond the minimal  $SU(5)$  require at least two independent scalar doublets for the *up* and *down* quark sectors, respectively.

[13] and thoroughly revised recently by Amaldi et al [9] after they have examined over 1600 split-multiplet combinations of quarks, leptons and scalars.

The ABC model is known [9] to be well consistent with present data and give perfect-single point unification when light split fermions are taken on the low mass scale near the TeV region while their heavy partners are on the grand one. It is easily comprehended on the other hand that such a situation requires a new special fine-tuning between the gigantic VEVs of the Higgs scalar in 1, 24 and 75 reps of the SU(5) to get in general the rather light split-multiplet fermions in the SU(5) model. Thus "staying alive with SU(5)" [13] seems to be even more problematic than the old hierarchy problem.

However, it seems to be quite reasonable to think that if one family of the split-multiplet fermions ( $n_F = 1$  in Eq.(2)) have to start rather early to correct a right way the running of the constants  $\alpha_1, \alpha_2$ , and  $\alpha_s$  two or three families of them could start later to do the same. Thus, we could expect that instead of one light family there appear two or would be better three (as for ordinary quarks and leptons) heavy families of split-multiplet fermions. Ideally, their mass scale  $M_{HSF}$  could be arranged at the "radiative distance" from the grand scale  $M_G$ ,  $M_{HSF} \sim \alpha_{GUT}^2 M_G$  so as not to have above mentioned hierarchy problem for the HSF spectrum. Fortunately, it happens to be the case in our model (see Table 1).

So, there appear at first time not only to derive the ABC Anzatz theoretically with our HSF scenario in the framework of the general SU(2n) GUTs but also to avoid the split fermion mass hierarchy problem introducing the several families of the HSF states. Considering the minimal possible GUT (n=3) we are led to SU(6) model with total fermion content (1,2) as

$$3 \cdot (2 \cdot \bar{6} + 15) + n_F \cdot (15 + \bar{15}) \quad (13)$$

containing only order-invariant part in Eq.(11).

Using then as input parameters the World average values  $\alpha_s(M_Z) = 0.117 \pm 0.005$ ,  $\alpha_{EM}(M_Z) = 1/(127.9 \pm 0.2)$  and  $\sin^2\theta_W(M_Z) = 0.2319 \pm 0.0008$  [14] in the standard RG

equations for the running constant  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_s$

$$\mu \frac{d}{d\mu} \alpha_i^{-1} = -\frac{1}{2\pi} \left( b_i + \frac{b_{ij}}{4\pi} \alpha_j + O(\alpha_i^2) \right) \quad (14)$$

with the 1-loop and 2-loop  $b$ -factors [15,9] we are driven at Table 1 showing a perfect single-point unification for different values of family number  $n_F$  of HSF states. Clearly, this picture will practically hold true in general case  $SU(2n)$  GUT as well.

Although, we used hitherto the electroweak angle value as input parameter to present the extended picture possible the model allows to predict in principle this angle itself if we start with three (or four) families of HSF states and consider the mass scale  $M_{HSF}$  of split fermions as of the pure radiative origin ( $M_{HSF} \sim \alpha_{GUT}^2 M_G$ ). So, the  $SU(2n)$  GUTs with three families of the complementary fermions seems to compare well with MSSM as to observable aspects of unification. However, the SUSY extension of the above  $SU(2n)$  theories are of special interest and we are coming to it now.

## 4 The HSF room in the SUSY $SU(2n)$ GUTs

Let us consider the minimal  $SU(6)$  model keeping in mind the whole class of the  $SU(2n)$  GUTs. The essential point related with the SUSY extension of those GUTs seems to be that they break themselves mainly along the foregoing  $SU(n)_S \otimes SU(n)_W \otimes U(I)$  channel providing thus the natural room for the HSF submultiplets.

The most general superpotential for the above heavy fields (chiral superfields now)  $\Phi_j^i$  and  $\varphi_i(\overline{\varphi}^i)$  in our  $SU(6)$  model looks as

$$W = \frac{\mu}{2} Tr \Phi^2 + \frac{h}{3} Tr \Phi^3 + m \overline{\varphi} \varphi + \lambda \overline{\varphi} \Phi \varphi \quad (15)$$

(the light Higgs supermultiplets  $H_i(\overline{H}^i)$  and  $H_{ij}(\overline{H}^{ij})$  of  $SU(6)$  generating masses of the *down* and *up* quarks (1), respectively, are not essential for the present discussion). The standard supersymmetric analysis of the F-terms of the heavy superfields  $\Phi$ ,  $\varphi$  and  $\overline{\varphi}$

$$F_\Phi = F_\varphi = F_{\overline{\varphi}} = 0 \quad (16)$$

during breaking process (3) shows that the trivial non-broken case apart the only symmetry breaking pattern of  $SU(6)$  in the no-scale limit of a superpotential  $W_0 = W(\mu = m = 0)$  is just the HSF channel  $SU(3) \otimes SU(3) \otimes U(I)$

$$\langle \Phi_j^i \rangle = \Lambda diag[1, 1, 1, -1, -1, -1]^i_j \quad \langle \varphi \rangle = \langle \bar{\varphi} \rangle = 0 \quad (17)$$

where the VEV parameter  $\Lambda$  is not fixed as yet. The switching on the masses  $\mu$  and  $m$  in  $W$  (15) does open the other channels as well leading among all the degenerate vacua to the familiar SM one with the VEVs of the scalars as

$$\begin{aligned} \langle \Phi_j^i \rangle &= \frac{m}{\lambda} diag[1, 1, 1, -1, -1, -1]^i_j + \frac{\mu}{h} diag[2, 2, 2, -3, -3, 0]^i_j \\ \langle \varphi_i \rangle &= \langle \bar{\varphi}^i \rangle = \left[ 6 \frac{\mu}{\lambda} \left( \frac{m}{\lambda} + \frac{\mu}{h} \right) \right]^{1/2} \delta_{i6} \end{aligned} \quad (18)$$

An interesting feature of the solution (17) seems to be that, while the supposedly largest rank-preserving part in the  $\Phi$  scalar VEV are determined by the mass parameter  $m$  of the fundamental scalar  $\varphi$ , in its own VEV  $\langle \varphi \rangle$  the contributions of the  $O(m)$  order are cancelled and the leading order is just the average geometrical one  $O(\sqrt{\mu m})$ ,  $m \gg \mu$ . So, there is an automatic  $SU(3) \otimes SU(3) \otimes U(I)$  intermediate gauge scale  $M_I = O(\sqrt{\mu m})$  in this case by contrast to the non-SUSY one (Sec.3) where we have (inspite of the dynamically appearing HSF scale) the single point gauge unification. However, as we can see below, this new scale is turned out also to be related with the HSF submultiplets.

The above SUSY analysis of the  $SU(6)$  symmetry breaking pattern remains in force after a typical low-energy SUSY breaking as well. The standard effective Higgs potential following from the minimal  $N = 1$  Supergravity [16]

$$V = \left| \frac{\partial W}{\partial z_i} + m_{3/2} z_i^* \right|^2 + m_{3/2}(A - 3)[W^* + W] + D - terms \quad (19)$$

can induce only the little shifts in the VEV in Eq.(18).of the gravitino masses order  $O(m_{3/2})$  at most. At the same time Supergravity could provide some reasoning for lifting vacuum degeneracy in favor of just the minimum (18) in the general case as well.

Now coming back to the supersymmetric analogue  $L_Y^{SUSY}$  of the Yukawa coupling (5), which continues to be invariant <sup>4</sup> under reflection (4) suppressing the direct mass term for the complementary fermions (and sfermions) (2), we are led from the VEV matrix  $\langle \Phi_j^i \rangle$  (16) to the above mass formula (9) adapted to our SU(6) case

$$\hat{M} = M_6 \hat{Y}_6 + M_5 \hat{Y}_5, \quad M_6 = G \frac{m}{\lambda} \sqrt{12}, \quad M_5 = G \frac{\mu}{h} \sqrt{60} \quad (20)$$

where  $G$  is Yukawa coupling constant and factors  $\sqrt{12}$  and  $\sqrt{60}$  appeared from the standard normalization of hypercharges  $Y_6$  and  $Y_5$ , respectively. It can be easily seen now that any gap between masses  $\mu$  and  $m$  of the scalars  $\Phi$  and  $\varphi$  are immediately transformed into the gap inside of the complementary particles in the rep  $15 + \overline{15}$  of SU(6) (see Eq.(13)) where the SUSY HSF submultiplets ( 3.3 ) + (  $\bar{3}\bar{3}$  ) have relatively light masses ( $M_{HSF} = \frac{\mu}{h}G$ ) while the remainders ( 3.1 ) + (  $\bar{3}.1$  ) + ( 1.3 ) + ( 1. $\bar{3}$  ) are much heavier ( $M_F = \frac{m}{\lambda}G = M_G G$ ,  $M_G$  is the unification mass) up to the grand scale ( $G \simeq 1$ ). According to Eq.(18) and (20) the  $SU(3) \otimes SU(3) \otimes U(I)$  intermediate scale  $M_I$  given by the VEV of the scalar  $\varphi(\bar{\varphi})$  can be expressed now through the basic parameters of the model - the unification mass  $M_G$  and the HSF scale  $M_{HSF}$

$$M_I = [M_G M_{HSF}]^{1/2} \cdot \eta, \quad \eta = \left( 6 \frac{h}{G\lambda} \right)^{1/2} \quad (21)$$

While in the ordinary case (Sec.3) the natural gap between masses  $M_{HSF}$  and  $M_G$  would be at most the radiative one now in the *SUSY* case mass  $M_{HSF}$  could be in principal anywhere below and even down to the SUSY scale.

One can understand, that in the framework of the ordinary MSSM giving the perfect single point unification [9] it would look quite hopeless to find any HSF states somewhere beyond the unification area itself. However, in our case of the dynamically stipulated in-

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<sup>4</sup>We could say that this invariance holds in the no-scale superpotential  $W_0$  as well in form of the *R*-parity

$$\Phi \rightarrow -\Phi, \quad \varphi(\bar{\varphi}) \rightarrow -\varphi(\bar{\varphi}), \quad W_0 = -W_0, \quad L_Y^{SUSY} = -L_Y^{SUSY}$$

what leads to the pure HSF vacuum (17). The mass terms in the general superpotential  $W$  (15) break this symmetry and induce the "scaled" solution (18) among the other degenerate ones.

termediate unification with the calculable scale (21) such a possibility appears even for the standard low-energy SUSY breaking strongly correlated with the electroweak scale, say  $M_{SUSY} = 3.2 \cdot 10^2 GeV$ . Our results are presented in Table 2 for one family of the HSF states and three different values of  $\alpha_s(M_Z)$  after the above RG equations (14) (with  $b_i$  and  $b_{ij}$  factors including contributions of the supersymmetric partners as well [15]) were numerically integrated in 2-loop level approximation. At the same time it seems to be of a special interest the question about the SUSY scale itself as following from the unification requirement only. The general dependence of the unification scale  $M_G$  (dashed line) and the HSF scale  $M_{HSF}$  (solid line) from the SUSY scale  $M_{SUSY}$  for the central value  $\alpha_s(M_Z) = 0.117$  are given on Fig.1 for one HSF family again (in  $\eta = 1$  case, see Eq.(21)). One can see clearly that the under-Plank mass unification requirement ( $M_G \leq M_{Pl}$ ) and general condition  $M_{HSF} \geq M_{SUSY}$  following from the effective  $N = 1$  Supergravity potential (19) leave the only possible area for the SUSY scale  $M_{SUSY} = 3 \cdot 10^2 \div 10^3 GeV$  and correspondingly for HSF scale  $M_{HSF} = 10^{11} \div 10^{16} GeV$ . The possible though not too restrictive limitation on the HSF room in this model could be expected from the  $b - \tau$  unification, stipulated by ordinary Yukawa couplings of quarks and leptons with the Higgs supermultiplets  $H_i(\overline{H}^j)$  and  $H_{ij}(\overline{H}^{ij})$  (see above). While we are going to discuss it closely in a separate publication we include in the Table 2 some b-quark mass 1-loop values  $m_b(m_b)$  taking in its RG equation the "maximal" top quark Yukawa constant on the unification scale  $Y_t(M_G) = 1$  ( $Y_t = G_t^2/a\pi$ ) and as top quark mass value  $m_t = 174 GeV$ . One can see that for the parameters involved somewhat large value of  $m_b$  appears for the Plank mass unification only.

So, we are driven at a conclusion that there could be some new physics in  $10^{11} - 10^{14} GeV$  region related with non-chiral extension of the MSSM. The above HSF particles affect the unification considerably increasing its scale to the string unification limit  $M_{Str} \approx 5 \cdot 10^{17} GeV$  and even up to Plank mass  $M_{Pl}$ . Simultaneously, they could lead to rather low value  $\alpha_s(M_Z)$  predicted from the low-energy physics [11] (see Table 2). In contrast to the non-SUSY

case the SUSY SU(6) (and SU(2n) GUTs in general) strongly prefer one HSF family case accommodating multi-family HSF states in the vicinity of the unification area itself ( $M_{HSF} \approx M_I \approx M_G$ ).

## 5 Summary

We have discussed as general as possible the problem of inclusion of the non-chiral matter in the SM and MSSM and found that there could naturally exist the special set of the relatively light HSF particles in the framework of the SU(2n) type GUTs. So far we knew only two "canonical" sets of the particles which ruled the unification phenomena in GUTs - the ordinary SM set with quarks, leptons and Higgs doublets, and the MSSM set including all their supersymmetric partners as well. The first set itself is turned out no to be enough to give the unification at all. Also the second set, the MSSM, while giving a perfect unification at  $M_G \approx 10^{16} GeV$  seems not to be enough to give the very desirable higher string unification at  $M_{Str} \approx 5 \cdot 10^{17} GeV$ .

The third set, derived here from the starting fermion spectrum (1,2) is the SM set plus HSF states (11), gives the perfect single point unification just at the same point  $M \approx 10^{16} GeV$  (see Table 1) as in the MSSM case. Thus, the HSF states works exactly like as SUSY partners of the SM particles for the running constants  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_S$ . Lastly, the fourth set which is just the SUSY version of the third one (SM + HSF) leads to the higher string and even Plank scale unification depending on the mass spectra of the HSF states. It seems to be quite interesting that in the both cases - ordinary and SUSY - HSF states naturally happen to be on the rather higher mass scale  $10^{11} - 10^{14} GeV$  (see Secs.3 and 4) and considerably modify the desert physics.

While at present there are no any direct indication in favor of the SU(2n) GUTs some arguments look to be relevant:

- (i) In contrast to the MSSM [11] strong coupling constant  $\alpha_S(M_Z)$  extrapolated

down from the unification scale meets the value extracted from the low-energy physics for ordinary (Sec.3, Table 1) as well as SUSY (Sec.4, Table 2) cases;

- (ii) In the higher  $SU(2n)$  symmetry cases  $SU(8)$ ,  $SU(10)$  etc. containing gauge family symmetries there could appear among the others the non-suppressed flavour-changing proton decay modes like as  $p \rightarrow \pi^0\mu^+, K^0e^+, \dots$
- (iii) An introducing of the complementary matter multiplets (2) in addition to the ordinary one of the SM or MSSM could help to resolve familiar vacuum  $\theta$ -domain problem [17] in a manner by Georgi and Wise [18].

We will consider all those and related problems elsewhere.

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**Table 1** The values of the split fermion scale  $M_{HSF}$ , unification scale  $M_G$  and the inversed unified constant  $\alpha_{GUT}^{-1}$  for different number of the HSF families ( $n_F = 1, 2, 3$ ) depending on  $\alpha_s = 0.117 \pm 0.005$ .

$n_F$	$M_{HSF}$ , GeV	$M_G$ , GeV	$\alpha_{GUT}^{-1}$
1	$4.6_{+30.1}^{-3.9} \cdot 10^3$	$9.1_{-4.2}^{+7.1} \cdot 10^{15}$	$35.4_{+0.6}^{-0.6}$
2	$6.0_{+6.9}^{-2.5} \cdot 10^9$	$8.5_{-3.9}^{+5.6} \cdot 10^{15}$	$35.5_{+0.5}^{-0.5}$
3	$7.1_{+8.9}^{-1.6} \cdot 10^{11}$	$7.9_{-3.4}^{+5.9} \cdot 10^{15}$	$35.5_{+0.5}^{-0.5}$

**Table 2** The values of the split supermultiplets scale  $M_{HSF}$ , unification scale  $M_G$  and the in inverted unified coupling constant  $\alpha_{GUT}^{-1}$  for one family HSF states depending on SUSY scale  $M_{SUSY} = 10^{2.5} GeV$  and different values of  $\alpha_s(M_Z)$ . The intermediate scale  $M_I$  is calculated to be  $M_I = 10^{15.2}$  when parameter  $\eta$  (see Eq.(21)) runs in some natural region  $0.5 \div 1.5$ . The 1-loop of b-quark mass values presented for "maximal" value of top quark Yukawa constant  $Y_t(M_G) = 1$  ( $m_t = 174 GeV$ ).

$\alpha_s(M_Z)$	$M_{HSF}$ , GeV	$M_G$ , GeV	$\alpha_{GUT}^{-1}$	$m_b(m_b)$ , GeV
0.112	$10^{13.6}$	$10^{17.4}$	19.2	4.32
0.117	$10^{12.0}$	$10^{18.4}$	15.6	5.17
0.122	$10^{10.7}$	$10^{19.2}$	12.5	5.66

## Figure Caption

**Fig.1** The general dependence of the unification scale  $M_G$  (dashed line) and HSF scale  $M_{HSF}$  (solid line) from the SUSY scale  $M_{SUSY}$  for the central value  $\alpha_s(M_Z) = 0.117$  (in  $\eta = 1$  case, see Eq.(21))